What is Pebbling?

Step 1: Find a node with two pebbles

What is Pebbling?

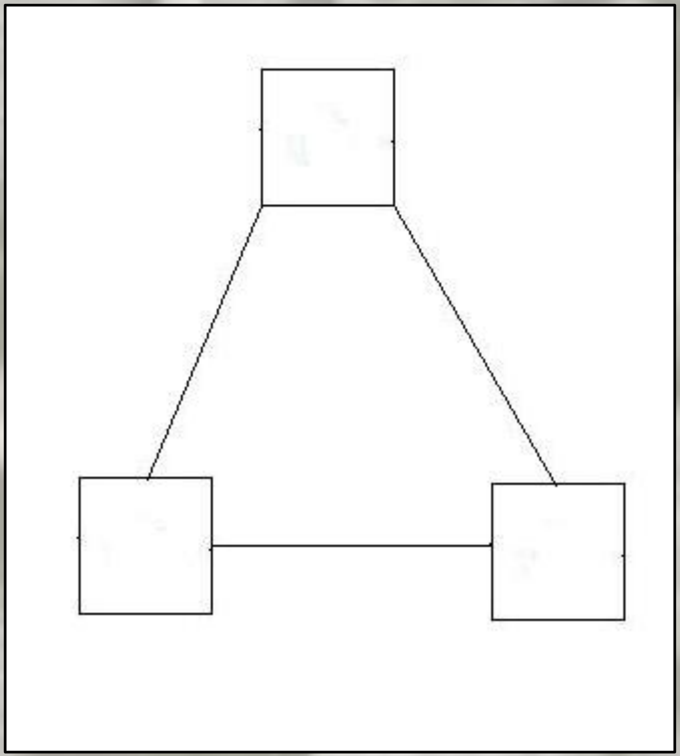
Step 2: Take two pebbles off of one vertex and place one pebble on an adjacent vertex. This is one pebbling move.

Pebbling Number ∏(G)

The pebbling number of a graph is the minimum number of pebbles such that for any arrangement of ∏(G) pebbles, any vertex can be reached by performing a set of pebbling moves.

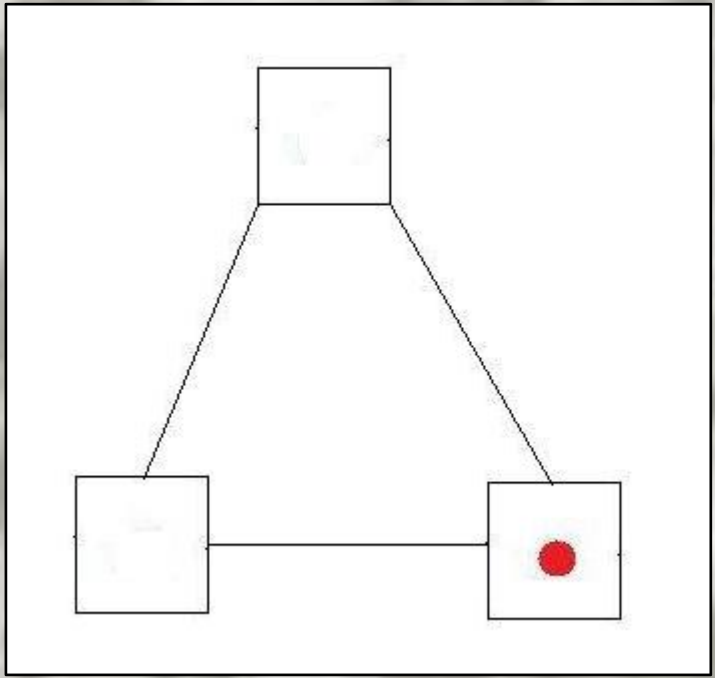
Example

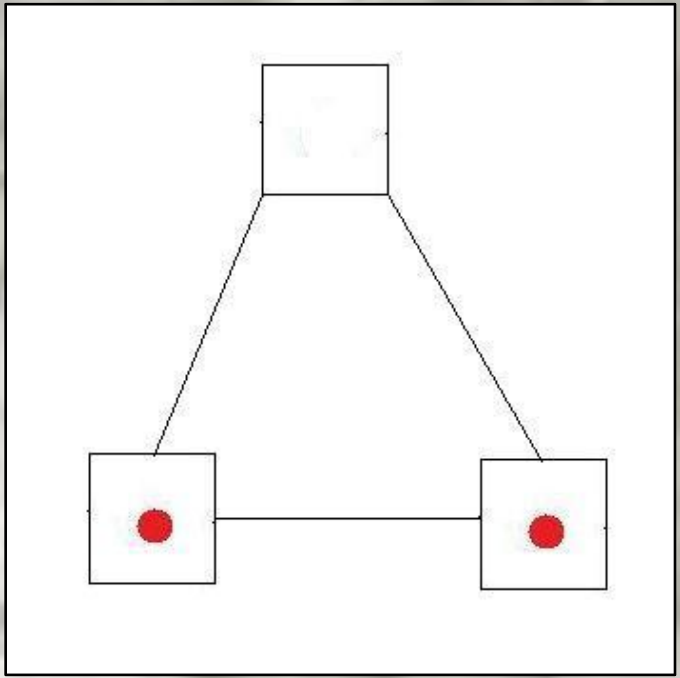
What is the pebbling number of this graph?



Example

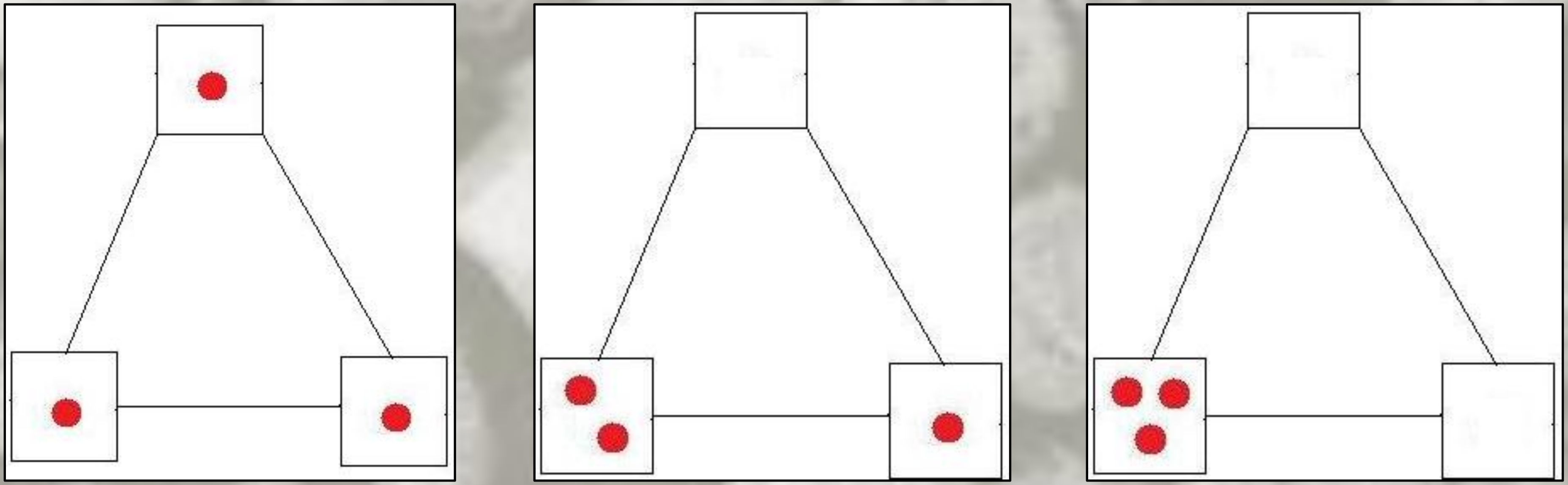
Neither one nor two is enough.





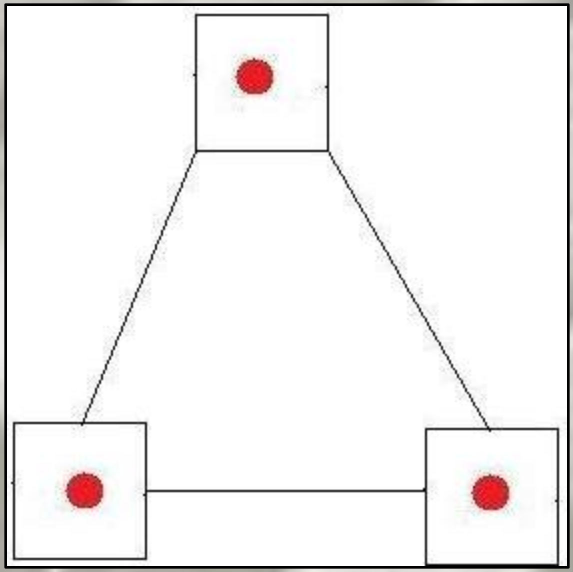
Example

Let’s try three. There are three unique ways to organize three pebbles on three nodes.



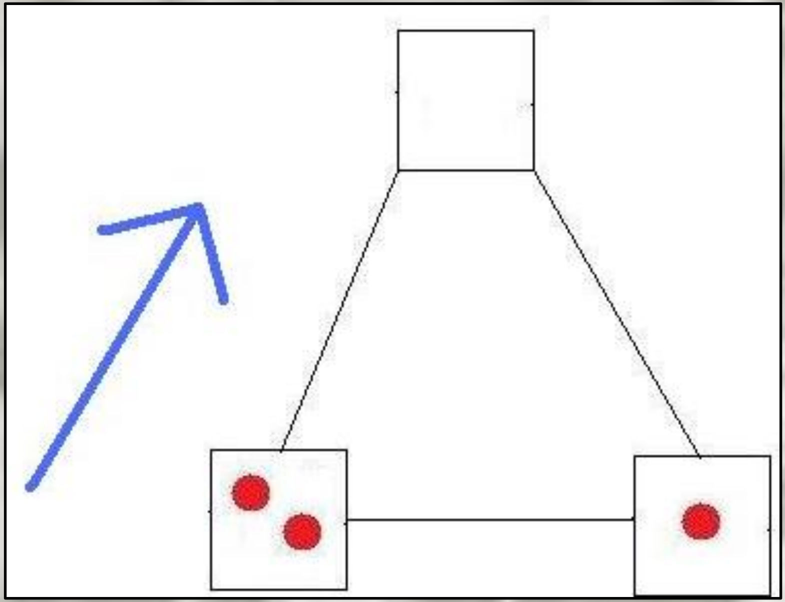
Example

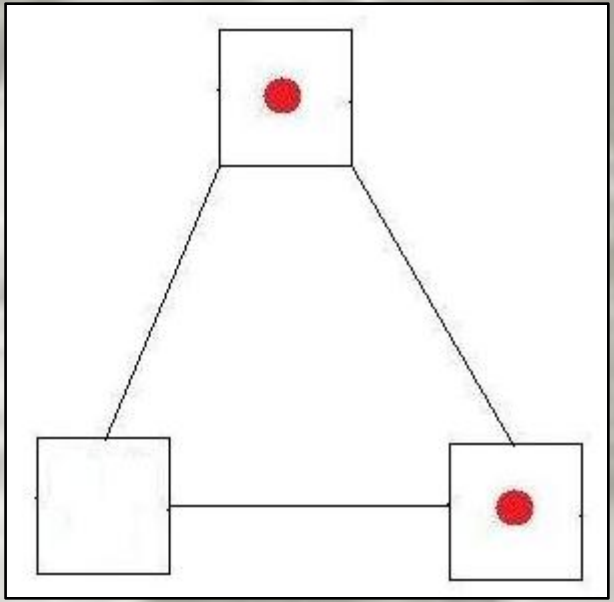
In option one, every node is already reached.



Example

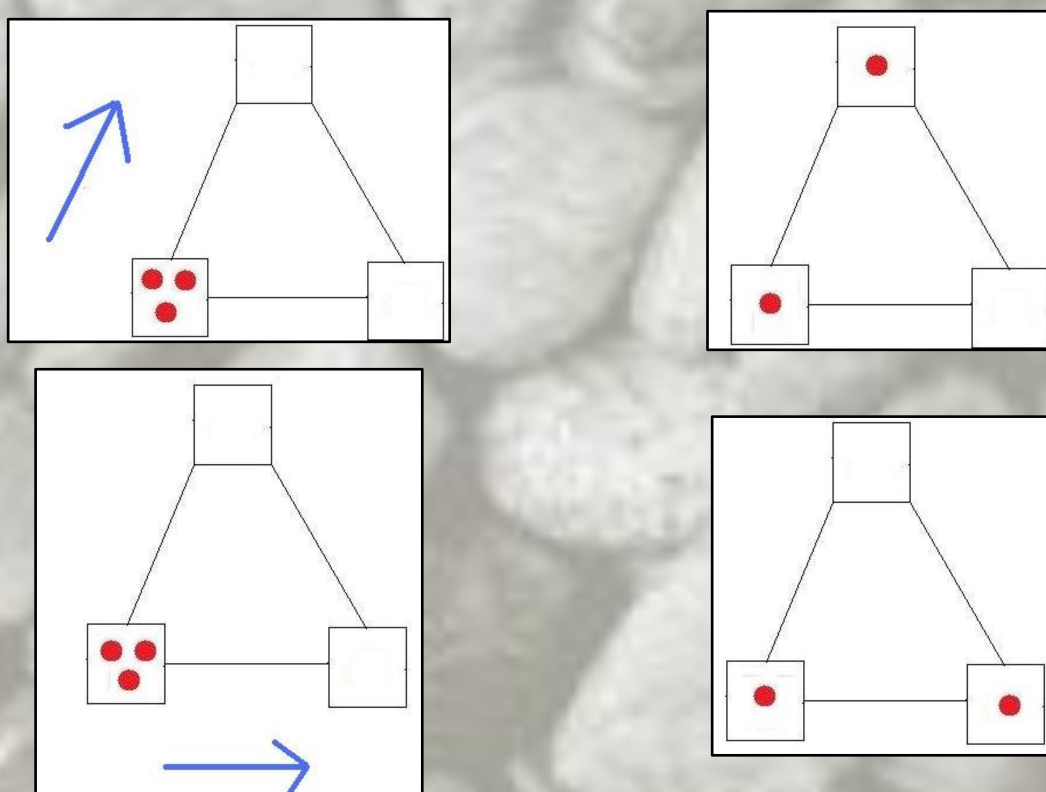
In option two, the only other node can be reached.





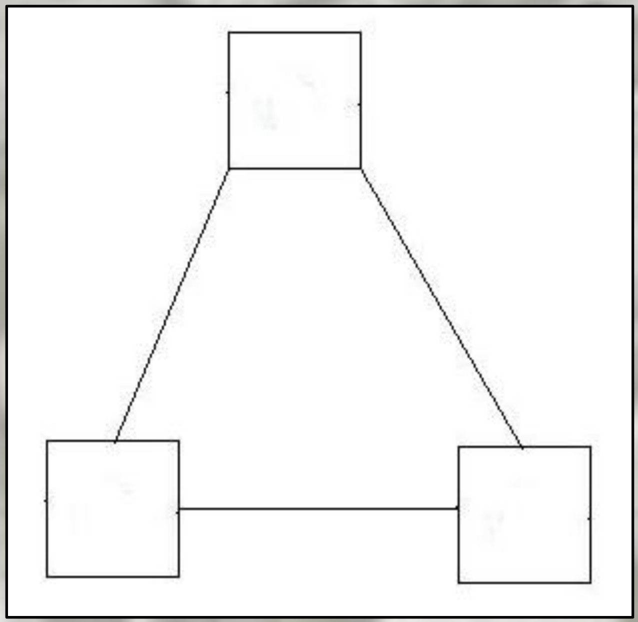
Example

In the third configuration, either node can be reached.



Example

Since for any configuration of three pebbles any node can be reached, $\prod(G) = 3$



Basic Pebbling Facts

- For any graph G, $\prod(G)>=V$, where V is the number of G's vertices.
- **Class-0 Graph –** A graph G such that $\prod(G)=V$

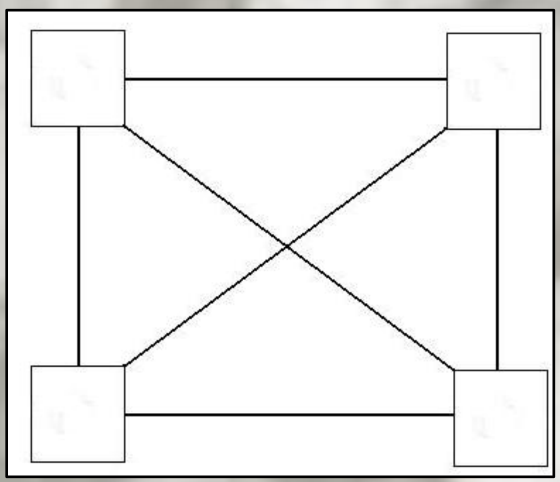
Partitioning

A partition of a graph is a set of subgraphs with three properties:

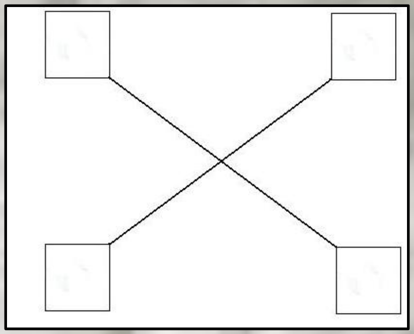
- Each subgraph has the same vertices as the original graph
- The union of the edges of the subgraphs is equal to the set of edges of the original graph
- No subgraphs have any edges in common

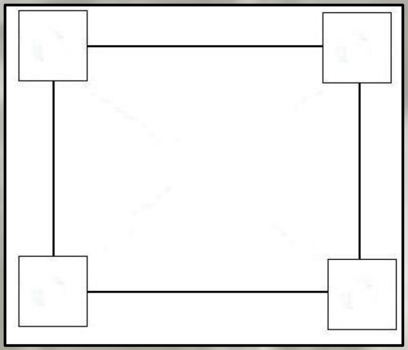
Example 1

Graph



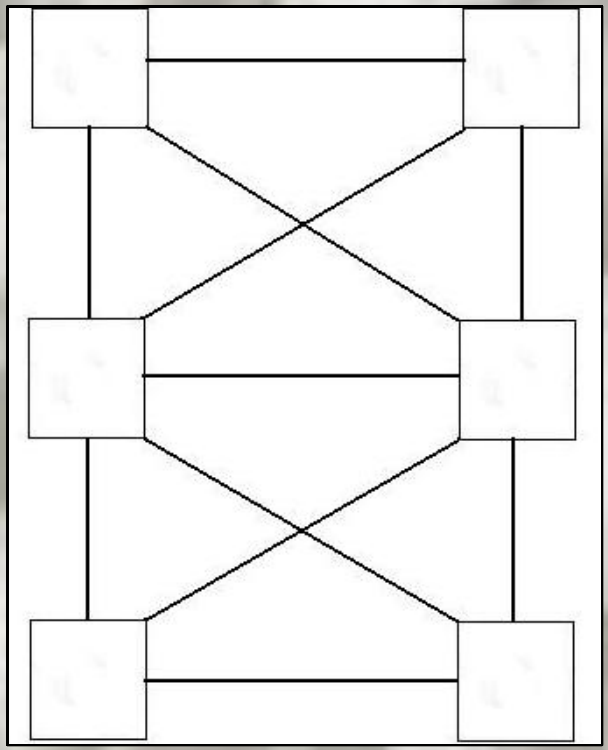
Partition



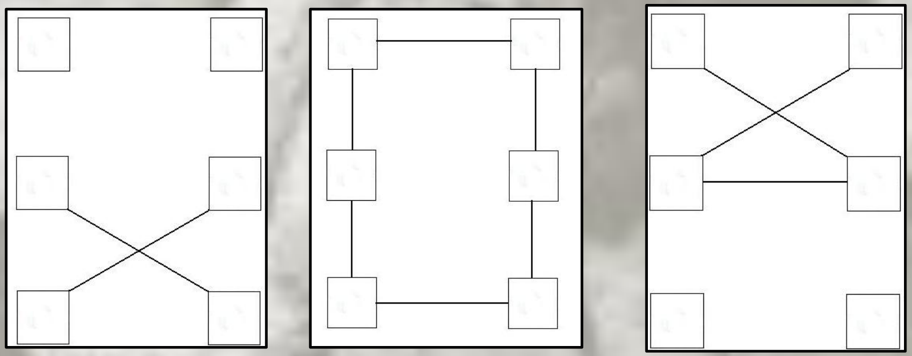


Example 2

Graph



Partition



K class-0 subgraphs

A simple graph G is k class-0 if G can be partitioned into k class-0 subgraphs.

Previous Result

For any positive integer k, there exists a positive integer N such that for all n>N a complete graph with n vertices is k class-0.

Research Question

For any positive integer k, does there exist a graph which is not complete which is k class-0?

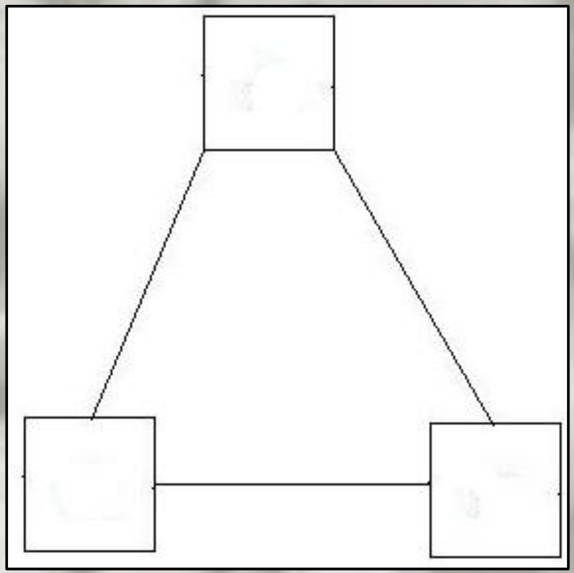
Graph Cross Product

The cross product of two graphs G and H, G x H, is constructed in the following way:

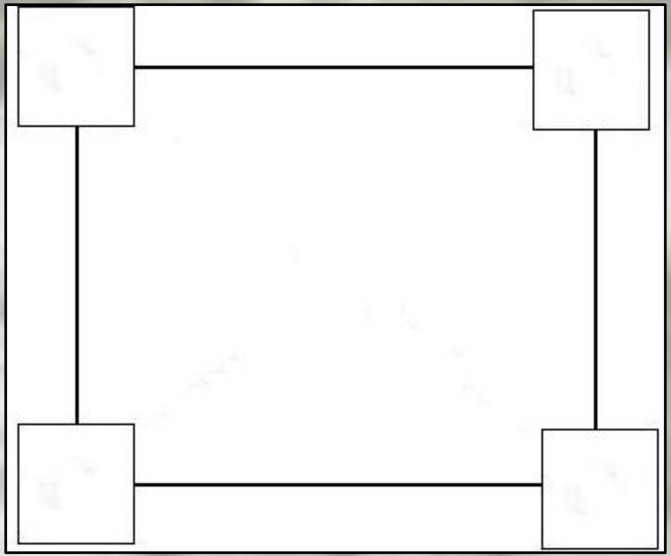
- For each vertex of G draw a graph of H. Let H_u denote the copy of H which replaced vertex u.
- For every edge (u, v) in G connect each vertex z in H_u to the corresponding z in H_v.

Example

T

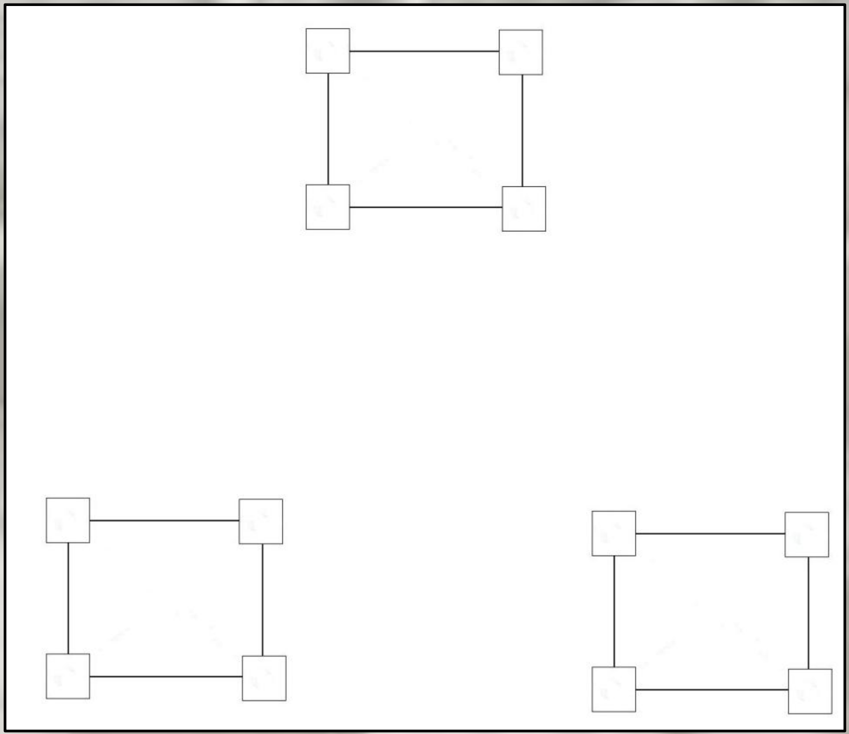


S



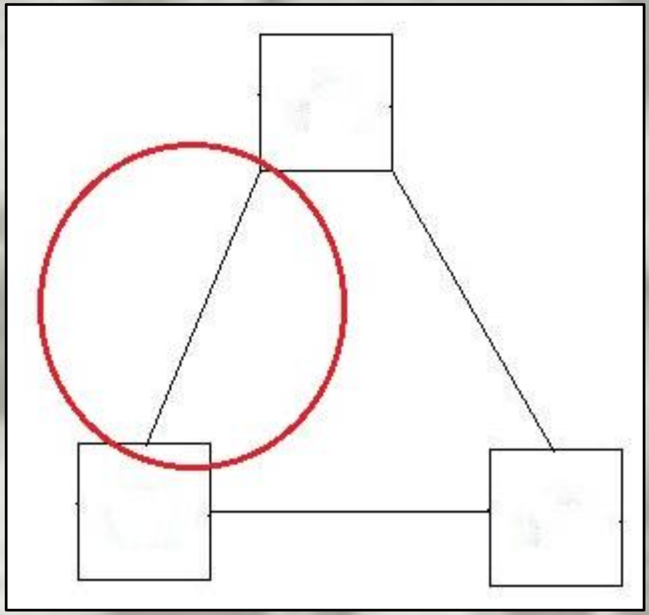
Example

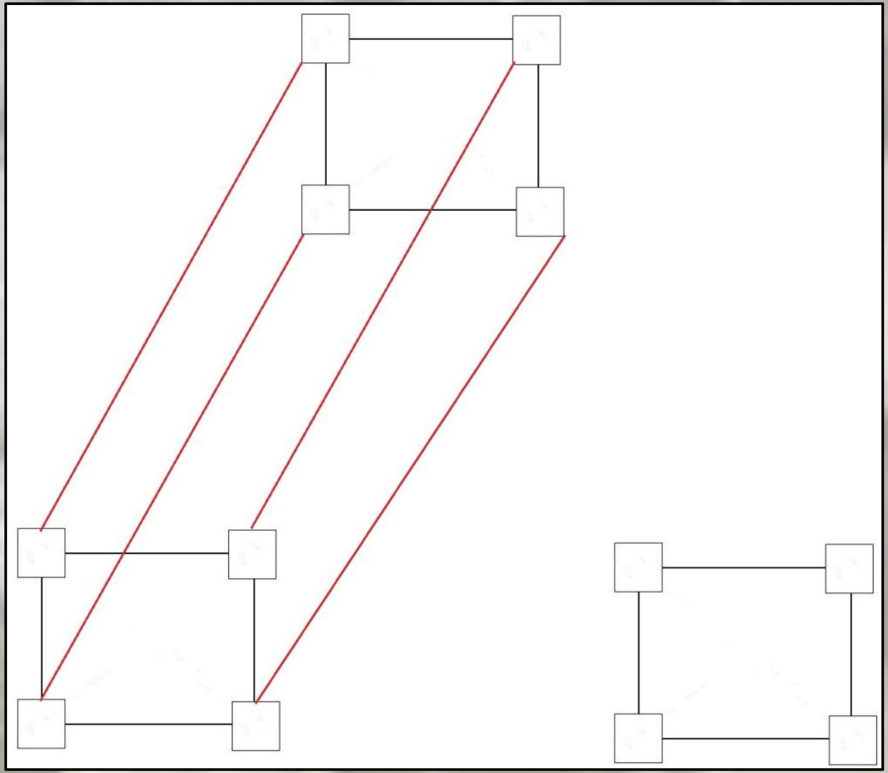
Start constructing T x S by replacing the vertices of T with copies of S.



Example

For each edge (u, v) in T connect corresponding vertices in S_u and S_v.





Example.

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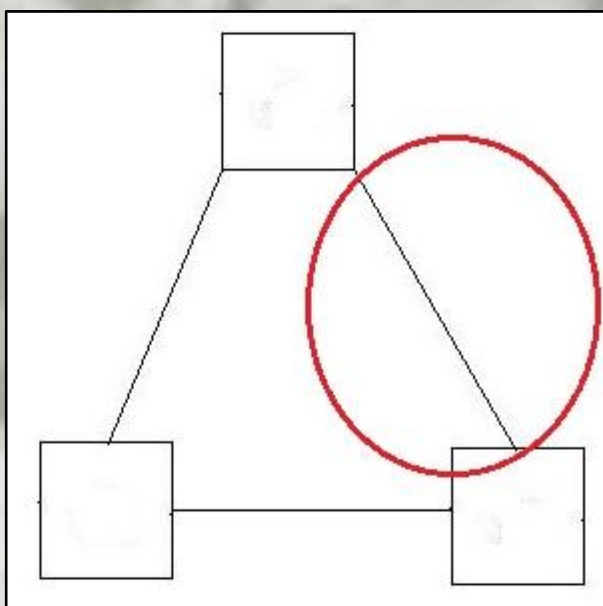

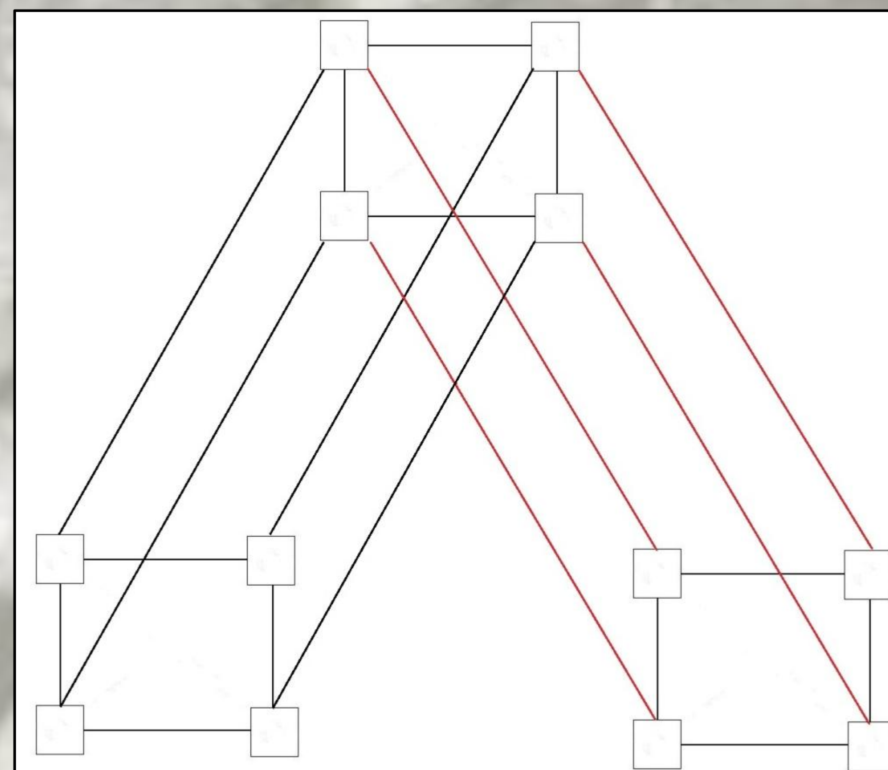

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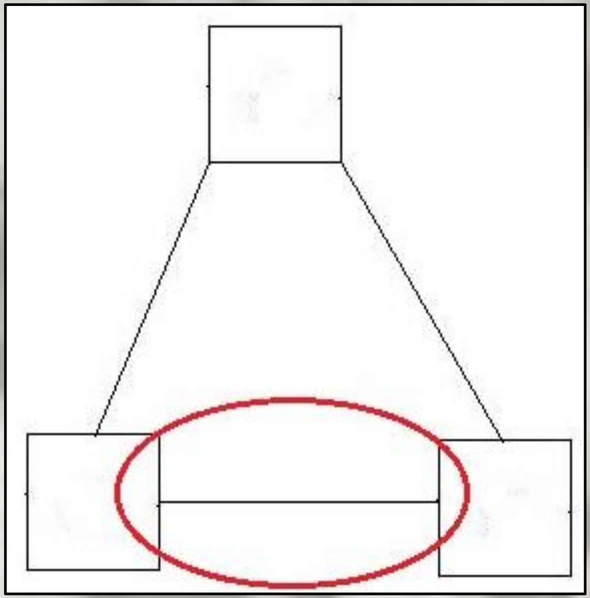
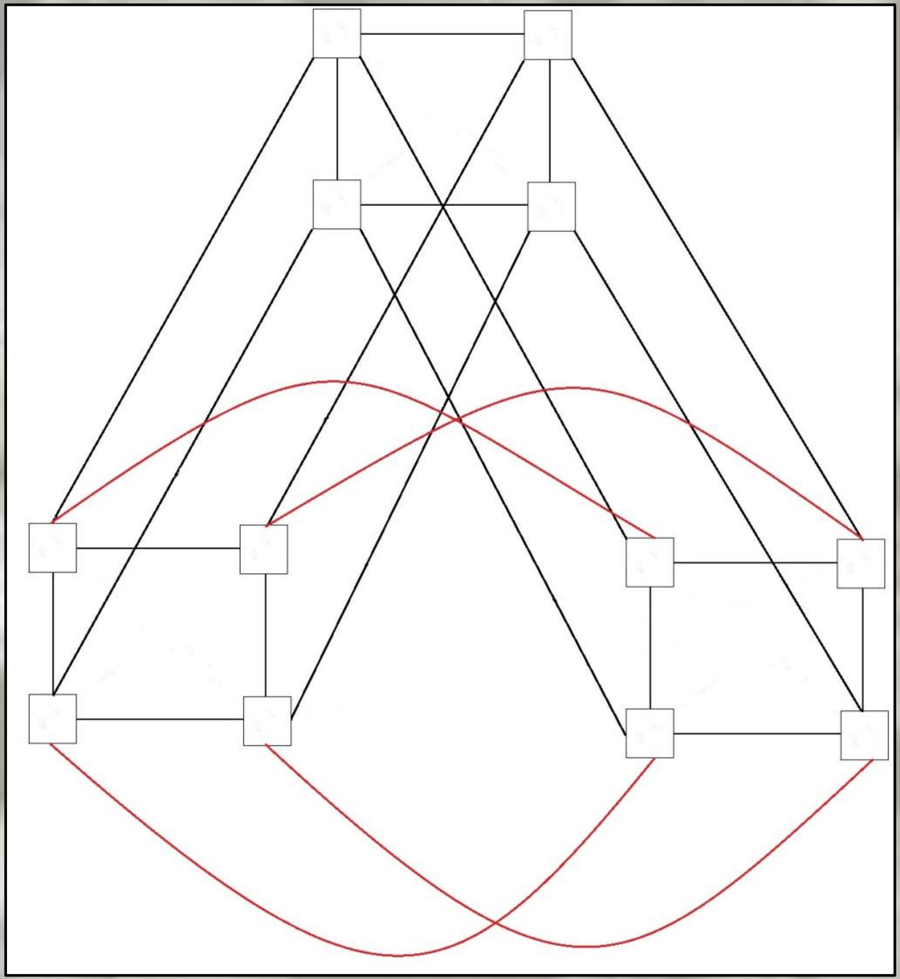
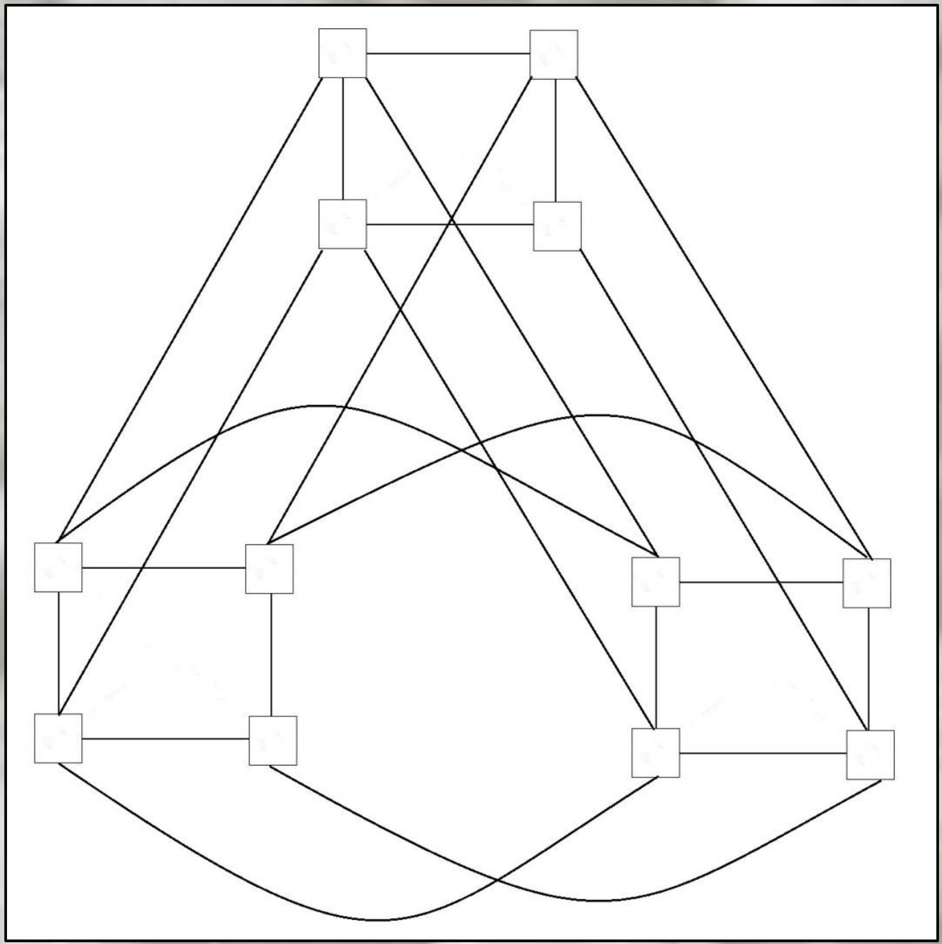




Example

- T x S=



Research Questions

- If graphs G and H are both class-0 are their products, G x H and H x G, also class-0?

- Does the following inequality hold for all graphs?
- Is it true for any certain types of graphs?

  $\prod(G \times H) <= \prod(G) \times \prod(H)$